1. Osnove merenja u digitalnim telekomunikacijama

Zadatak 1. Izvesti izraze za sigurne granice relativnih i apsolutnih grešaka indirektno merenih veličina

$$y_1 = C \cdot x_1 \tag{1.1}$$
$$y_2 = x_1 + x_2 \tag{1.2}$$
$$y_3 = x_1 - x_2 \tag{1.3}$$
$$y_4 = x_1 \cdot x_2 \tag{1.4}$$
$$y_5 = x_1 / x_2 \tag{1.5}$$
$$y_6 = x_1^2 \tag{1.6}$$
$$y_7 = \sqrt{x_1} \tag{1.7}$$
$$y_8 = f(x_1) \tag{1.8}$$

Gde su C - konstanta, x_1 i x_2 – direktno merene veličine, Gx_1 i Gx_2 - apsolutne greške merenja veličina x_1 i x_2 respektivno, a $f(x)$ – neprekidna, diferencijabilna funkcija.

Rešenje

Prema definiciji, izraz za sigurne granice relativne greške merenja veličine

$$y = f(x_1, x_2, \ldots, x_n) \tag{1.9}$$

je

$$\left|\Gamma_y\right| = \left|\frac{Gy}{y}\right| \le \sum_{i=1}^{n} \left|\frac{d}{dx_i} \ln\left[f(x_1, x_2, \ldots, x_n)\right]\right| \cdot \left|Gx_i\right| \tag{1.10}$$

dok je apsolutna greška data sa

$$G_y = Gy = y \cdot \Gamma_y \tag{1.11}$$

Prema tome, sigurne granice relativnih I apsolutnih grešaka traženih funkcija iznose

$$\left|\frac{Gy_1}{y_1}\right| \le \left|\frac{d}{dx_1} \ln(C \cdot x_1)\right| \cdot \left|Gx_1\right| = \left|\frac{d}{dx_1}(\ln C + \ln x_1)\right| \cdot \left|Gx_1\right| = \left|\frac{Gx_1}{x_1}\right| \tag{1.12}$$

$$\left|Gy_1\right| \le \left|\frac{Gx_1}{x_1}\right| \cdot \left|y_1\right| = \left|\frac{Gx_1}{x_1}\right| \cdot \left|C \cdot x_1\right| = \left|C\right| \cdot \left|Gx_1\right| \tag{1.13}$$

$$\left|\frac{Gy_2}{y_2}\right| \le \left|\frac{d}{dx_1}\ln(x_1+x_2)\right|\cdot|Gx_1| + \left|\frac{d}{dx_2}\ln(x_1+x_2)\right|\cdot|Gx_2| = \left|\frac{Gx_1}{x_1+x_2}\right| + \left|\frac{Gx_2}{x_1+x_2}\right| \tag{1.14}$$

$$\begin{aligned}|Gy_2| &\le \left|\frac{Gx_1}{x_1+x_2}\right|\cdot|y_2| + \left|\frac{Gx_2}{x_1+x_2}\right|\cdot|y_2| = \left|\frac{Gx_1}{x_1+x_2}\right|\cdot|x_1+x_2| + \left|\frac{Gx_2}{x_1+x_2}\right|\cdot|x_1+x_2| = \\ &= |Gx_1| + |\Delta x_2|\end{aligned} \tag{1.15}$$

$$\left|\frac{Gy_3}{y_3}\right| \le \left|\frac{d}{dx_1}\ln(x_1-x_2)\right|\cdot|Gx_1| + \left|\frac{d}{dx_2}\ln(x_1-x_2)\right|\cdot|Gx_2| = \left|\frac{Gx_1}{x_1-x_2}\right| + \left|\frac{Gx_2}{x_1-x_2}\right| \tag{1.16}$$

$$\begin{aligned}|Gy_3| &\le \left|\frac{Gx_1}{x_1-x_2}\right|\cdot|y_3| + \left|\frac{Gx_2}{x_1-x_2}\right|\cdot|y_3| = \left|\frac{Gx_1}{x_1-x_2}\right|\cdot|x_1-x_2| + \left|\frac{Gx_2}{x_1-x_2}\right|\cdot|x_1-x_2| = \\ &= |Gx_1| + |Gx_2|\end{aligned} \tag{1.17}$$

$$\begin{aligned}\left|\frac{Gy_4}{y_4}\right| &\le \left|\frac{d}{dx_1}\ln(x_1\cdot x_2)\right|\cdot|Gx_1| + \left|\frac{d}{dx_2}\ln(x_1\cdot x_2)\right|\cdot|Gx_2| = \\ &= \left|\frac{d}{dx_1}(\ln x_1+\ln x_2)\right|\cdot|Gx_1| + \left|\frac{d}{dx_2}(\ln x_1+\ln x_2)\right|\cdot|Gx_2| = \\ &= \left|\frac{Gx_1}{x_1}\right| + \left|\frac{Gx_2}{x_2}\right|\end{aligned} \tag{1.18}$$

$$\begin{aligned}|Gy_4| &\le \left|\frac{Gx_1}{x_1}\right|\cdot|y_4| + \left|\frac{Gx_2}{x_2}\right|\cdot|y_4| = \left|\frac{Gx_1}{x_1}\right|\cdot|x_1\cdot x_2| + \left|\frac{Gx_2}{x_2}\right|\cdot|x_1\cdot x_2| = \\ &= |Gx_1|\cdot|x_2| + |Gx_2|\cdot|x_1|\end{aligned} \tag{1.19}$$

$$\begin{aligned}\left|\frac{Gy_5}{y_5}\right| &\le \left|\frac{d}{dx_1}\ln(x_1/x_2)\right|\cdot|Gx_1| + \left|\frac{d}{dx_2}\ln(x_1/x_2)\right|\cdot|Gx_2| = \\ &= \left|\frac{d}{dx_1}(\ln x_1-\ln x_2)\right|\cdot|Gx_1| + \left|\frac{d}{dx_2}(\ln x_1-\ln x_2)\right|\cdot|Gx_2| = \\ &= \left|\frac{Gx_1}{x_1}\right| + \left|\frac{Gx_2}{x_2}\right|\end{aligned} \tag{1.20}$$

$$\begin{aligned}|Gy_5| &\le \left|\frac{Gx_1}{x_1}\right|\cdot|y_5| + \left|\frac{Gx_2}{x_2}\right|\cdot|y_5| = \left|\frac{Gx_1}{x_1}\right|\cdot|x_1/x_2| + \left|\frac{Gx_2}{x_2}\right|\cdot|x_1/x_2| = \\ &= \frac{|Gx_1|}{|x_2|} + |Gx_2|\cdot\frac{|x_1|}{x_2^2}\end{aligned} \tag{1.21}$$

$$\left|\frac{Gy_6}{y_6}\right| \le \left|\frac{d}{dx_1}\ln\left(x_1^2\right)\right| \cdot \left|Gx_1\right| = \left|\frac{d}{dx_1}\left(2\ln x_1\right)\right| \cdot \left|Gx_1\right| = 2\left|\frac{d}{dx_1}\left(\ln x_1\right)\right| \cdot \left|Gx_1\right| = 2\left|\frac{Gx_1}{x_1}\right| \tag{1.22}$$

$$\left|Gy_6\right| \le 2\left|\frac{Gx_1}{x_1}\right| \cdot \left|y_6\right| = 2\left|\frac{Gx_1}{x_1}\right| \cdot x_1^2 = 2\left|Gx_1\right| \cdot \left|x_1\right| \tag{1.23}$$

$$\left|\frac{Gy_7}{y_7}\right| \le \left|\frac{d}{dx_1}\ln\left(\sqrt{x_1}\right)\right| \cdot \left|Gx_1\right| = \left|\frac{d}{dx_1}\left(\frac{1}{2}\ln x_1\right)\right| \cdot \left|Gx_1\right| = \frac{1}{2}\left|\frac{d}{dx_1}\left(\ln x_1\right)\right| \cdot \left|Gx_1\right| = \frac{1}{2}\frac{\left|Gx_1\right|}{x_1} \tag{1.24}$$

$$\left|Gy_7\right| \le \frac{1}{2}\frac{\left|Gx_1\right|}{x_1} \cdot \left|y_7\right| = \frac{1}{2}\frac{\left|Gx_1\right|}{x_1} \cdot \sqrt{x_1} = \frac{1}{2}\frac{\left|Gx_1\right|}{\sqrt{x_1}} \tag{1.25}$$

$$\left|\frac{Gy_8}{y_8}\right| \le \left|\frac{d}{dx_1}\ln\left[f\left(x_1\right)\right]\right| \cdot \left|Gx_1\right| = \left|\frac{1}{f(x)} \cdot \frac{d}{dx_1}f(x)\right| \cdot \left|Gx_1\right| = \left|\frac{f'(x)}{f(x)}\right| \cdot \left|Gx_1\right| \tag{1.26}$$

$$\left|Gy_8\right| \le \left|\frac{f'(x)}{f(x)}\right| \cdot \left|Gx_1\right| \cdot \left|y_8\right| = \left|\frac{f'(x)}{f(x)}\right| \cdot \left|Gx_1\right| \cdot \left|f(x)\right| = \left|f'(x)\right| \cdot \left|Gx_1\right| \tag{1.27}$$

Zadatak 2. Izvesti izraze za sigurne granice relativne grešake indirektno merene veličine

$$y = \frac{x_1 x_2}{\sqrt{x_3^2 + x_4^2}} \tag{2.1}$$

Gde su x_1, x_2, x_3 i x_4 – direktno merene veličine, a $|Gx_1|$, $|Gx_2|$, $|Gx_3|$ i $|Gx_4|$ - njihove apsolutne greške merenja.

Rešenje

Koristeći relacije iz zadatka 1 dobija se da je

$$\begin{aligned}\left|\Gamma_y\right| &= \left|\Gamma_{\frac{x_1x_2}{\sqrt{x_3^2+x_4^2}}}\right| \le \left|\Gamma_{x_1x_2}\right| + \left|\Gamma_{\sqrt{x_3^2+x_4^2}}\right| \le \left|\Gamma_{x_1}\right| + \left|\Gamma_{x_2}\right| + \left|\Gamma_{\sqrt{x_3^2+x_4^2}}\right| \le \left|\Gamma_{x_1}\right| + \left|\Gamma_{x_2}\right| + \frac{1}{2}\left|\Gamma_{x_3^2+x_4^2}\right| \le \\ &\le \left|\Gamma_{x_1}\right| + \left|\Gamma_{x_2}\right| + \frac{1}{2}\left|\frac{x_3^2}{x_3^2+x_4^2}\right|\left|\Gamma_{x_3^2}\right| + \frac{1}{2}\left|\frac{x_4^2}{x_3^2+x_4^2}\right|\left|\Gamma_{x_4^2}\right| \le \\ &\le \left|\Gamma_{x_1}\right| + \left|\Gamma_{x_2}\right| + \left|\frac{x_3^2}{x_3^2+x_4^2}\right|\left|\Gamma_{x_3}\right| + \left|\frac{x_4^2}{x_3^2+x_4^2}\right|\left|\Gamma_{x_4}\right|\end{aligned} \tag{2.2}$$

Zadatak 3. Izvesti izraze za sigurne granice relativne grešake indirektno merene veličine

$$y = A\cos(\omega x) \tag{3.1}$$

Gde su A i ω konstante, x – direktno merena veličina, a $|\Delta x|$ - njena apsolutna greška merenja.

Rešenje

Kod složenijih formula, preporuka je izvoditi izraze za sigurne granice relativne greške po definiciji. Koristeći relaciju 1.10 dobija se da je

$$\begin{aligned}\left|\Gamma_y\right| &\leq \left|\frac{d}{dx}\ln\left[A\cos(\omega x)\right]\right|\cdot|Gx| = \left|\frac{d}{dx}\left[\ln A + \ln\cos(\omega x)\right]\right|\cdot|Gx| = \left|\frac{d}{dx}\ln\cos(\omega x)\right|\cdot|Gx| = \\ &= \left|\omega\frac{1}{\cos(\omega x)}\left(-\sin(\omega x)\right)\right|\cdot|Gx| = \left|\mathrm{tg}(\omega x)\right|\cdot|\omega\cdot Gx|\end{aligned} \tag{3.2}$$

Zadatak 4. Potrebno je projektovati pristupnu ADSL mrežu konfiguracije tačka-tačka. Električna snaga ubačena u liniju od strane predajnika iznosi P_t = 1 mW. Prag osetljivosti prijemnika je a_r = –35 dBm. Slabljenje konektora na prijemu i predaji je 2 dB. Rezerva sistema treba da bude 10 dB. Odrediti maksimalnu dužinu veze, ako se koristi ADSL kabel podužnog slabljenja 5 dB/km

Rešenje

Snaga predjnika izražena u dBm iznosi

$$a_t = 10\log\frac{1\,\mathrm{mW}}{1\,\mathrm{mW}} = 0\ \mathrm{dBm} \tag{4.1}$$

Ukupno slabljenje u sistemu mora biti manje od razlike snaga predajnika i osetljivosti prijemnika uz zadržavanje potrebne rezerve:

$$a = 2\ \mathrm{dB} + l\cdot 5\,\frac{\mathrm{dB}}{\mathrm{km}} + 2\ \mathrm{dB} < a_t - a_r - a_{rez} \tag{4.2}$$

Odavde se dobija da je dužina linije

$$\begin{aligned} l &< \frac{a_t - a_r - a_{rez} - 4\ \mathrm{dB}}{5\ \mathrm{dB/km}} = \frac{0\ \mathrm{dBm} - (-35\ \mathrm{dBm}) - 10\ \mathrm{dB} - 4\ \mathrm{dB}}{5\ \mathrm{dB/km}} = \\ &= \frac{35\ \mathrm{dB} - 14\ \mathrm{dB}}{5\ \mathrm{dB/km}} = 4{,}2\ \mathrm{km}\end{aligned} \tag{4.3}$$